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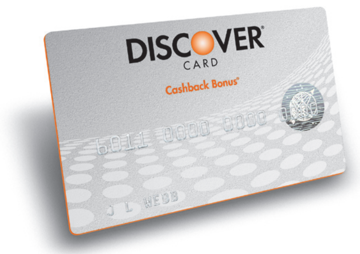

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C H A P T E R

6 Oblique Triangles and Vectors

In this chapter we examine several applications of the trigonometric functions. We begin with the law of sines and the law of cosines. These are theorems that permit the solution of triangles which are not right triangles. (Recall that we examined the solution of right triangles in section 1–3.) We then introduce the subject of vectors, which has wide application in science and engineering, and which, in a generalized form called linear algebra, has applications in the social and economic sciences as well.

6–1 The law of sines

A triangle in which none of the angles is a right angle is called an **oblique** triangle. One method for solving certain oblique triangles is the **law of sines.** The following paragraphs develop this law.

First, we observe that *at least two of the angles in every triangle are acute.* If only one angle were acute (less than 90°) then the other two would be obtuse or right (greater than or equal to 90°). This is impossible since the sum of these two angles would be greater than or equal to 180°, and thus the sum of all three angles would be greater than 180°.

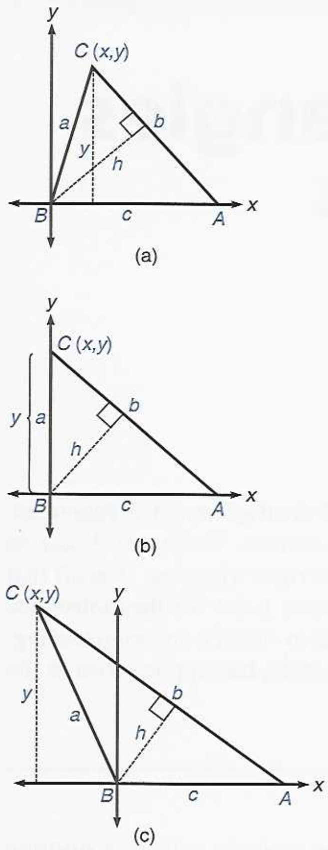


Figure 6–1

Now we consider any triangle ABC, and label two of the acute angles A and C. Angle B may be acute, obtuse, or right. We place the triangle in a coordinate system so that angle B is in standard position. Figure 6–1 shows sketches for the cases where B is (a) acute, (b) right, and (c) obtuse.

From vertex B we construct a line segment perpendicular to side AC and label this line h. From what we know about right triangles we can see that, in all three cases,

$$\sin A = \frac{h}{c} \quad \text{and} \quad \sin C = \frac{h}{a}$$

If we solve for h in each, we obtain

$$h = c \sin A \quad \text{and} \quad h = a \sin C$$

Since $c \sin A$ and $a \sin C$ equal the same quantity (h) they themselves must be equal. Thus,

$$c \sin A = a \sin C$$

$$\frac{c \sin A}{ac} = \frac{a \sin C}{ac} \qquad \text{Divide both members by } ac$$

$$\frac{\sin A}{a} = \frac{\sin C}{c} \qquad \text{Remove common factors}$$

We now extend the relation above to include angle B and side b. Let (x,y) be the coordinates of the vertex of angle C. From what we know about the trigonometric functions for any angle in standard position (chapter 2), we see that

$$\sin B = \frac{y}{a} \quad \text{or} \quad y = a \sin B$$

From what we know about right triangles,

$$\sin A = \frac{y}{b} \quad \text{or} \quad y = b \sin A$$

Thus, $a \sin B = y = b \sin A$, so

$$a \sin B = b \sin A$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

Putting these results together we have the law of sines.

The law of sines

In any triangle ABC,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Concept

The ratio of the sine of an angle to the length of the side opposite that angle is the same for all angles in any triangle.

Note We only use two of the three ratios at a time.

In the rest of this chapter we always assume side a is opposite angle A, side b is opposite angle B, and side c is opposite angle C.

■ ***Example 6–1 A***

Solve the triangle ABC. Round off answers to tenths.

$a = 13.2, A = 21.3°, B = 61.4°$

It is a good idea to make a table of values:

a: 13.2	A: 21.3°
b: ?	B: 61.4°
c: ?	C: ?

We can find C first.

$$\begin{aligned} C &= 180° - A - B \\ &= 180° - 21.3° - 61.4° \\ &= 97.3° \end{aligned}$$

The sum of the measure of all three angles is 180°

Now we fill in the law of sines.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin 21.3°}{13.2} = \frac{\sin 61.4°}{b} = \frac{\sin 97.3°}{c}$$

To use the law of sines we must always know one of the three ratios completely. In this case, we know the first ratio, so we use it to solve the other two.

Using the first and second ratios:

$$\frac{\sin 21.3°}{13.2} = \frac{\sin 61.4°}{b}$$

$$b \sin 21.3° = 13.2 \sin 61.4° \quad \text{Multiply each member by } 13.2b$$

$$b = \frac{13.2 \sin 61.4°}{\sin 21.3°} \approx 31.9 \quad \text{Divide each member by } \sin 21.3°$$

Using the first and third ratios:

$$\frac{\sin 21.3°}{13.2} = \frac{\sin 97.3°}{c}$$

$$c \sin 21.3° = 13.2 \sin 97.3° \quad \text{Multiply each member by } 13.2c$$

$$c = \frac{13.2 \sin 97.3°}{\sin 21.3°} \approx 36.0 \quad \text{Divide each member by } \sin 21.3°$$

Thus we have solved the triangle:

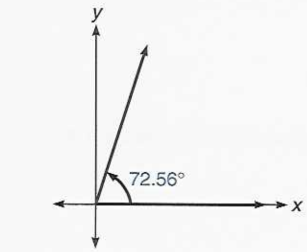
a: 13.2	A: 21.3°
b: 31.9	B: 61.4°
c: 36.0	C: 97.3°.

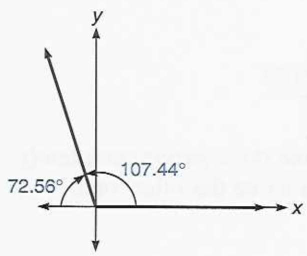
■

The ambiguous case

If we are given only one of the two angles of a triangle it is possible to get two different solutions to the problem. The reason for this is shown in example 6–1 B. When we use the law of sines to solve a triangle *for which only one angle is known* we call this the **ambiguous case.**

■ *Example 6–1 B*





Solve each triangle. Round off answers to tenths.

1. $a = 28.5, b = 30.0, A = 65°$.

a: 28.5	A: 65°	Make a table of values
b: 30.0	B: ?	
c: ?	C: ?	

$$\frac{\sin 65°}{28.5} = \frac{\sin B}{30.0} = \frac{\sin C}{c}$$ Fill values into the law of sines

$$\frac{\sin 65°}{28.5} = \frac{\sin B}{30.0}$$ Use the first two ratios to find angle B

$$30.0 \sin 65° = 28.5 \sin B$$

$$\frac{30.0 \sin 65°}{28.5} = \sin B$$

A *reference angle* for angle B is found with the inverse sine function.

$$B' = \sin^{-1}\left(\frac{30.0 \sin 65°}{28.5}\right) \approx 72.56°$$

Since angle B is in a triangle, we know its measure is between 0° and 180°. Thus, using B' as a reference angle B could be either 72.56° or $180° - 72.56° = 107.44°$. See the figure.

At this point we must divide the problem into two cases: the case where $B \approx 72.56°$ and the one where $B \approx 107.44°$.

Case 1: $B \approx 72.56°$

a: 28.5	A: 65°
b: 30.0	B: 72.56°
c: ?	C: ?

$$C \approx 180° - 65° - 72.56° \approx 42.44°$$

We can use the value of angle C to find c.

$$\frac{\sin 65^\circ}{28.5} \approx \frac{\sin 42.44^\circ}{c}$$

$$c \sin 65^\circ \approx 28.5 \sin 42.44^\circ$$

$$c \approx \frac{28.5 \sin 42.44^\circ}{\sin 65^\circ} \approx 21.2$$

Thus, $C \approx 42.4^\circ$, $c \approx 21.2$.

Case 2: $B \approx 107.44^\circ$

a: 28.5	A: 65°
b: 30.0	B: 107.44°
c: ?	C: ?

$$C \approx 180^\circ - 65^\circ - 107.44^\circ \approx 7.56^\circ$$

$$\frac{\sin 65^\circ}{28.5} \approx \frac{\sin 7.56^\circ}{c}$$

$$c \sin 65^\circ \approx 28.5 \sin 7.56^\circ$$

$$c \approx \frac{28.5 \sin 7.56^\circ}{\sin 65^\circ} \approx 4.1$$

Thus, $C \approx 7.6^\circ$, $c \approx 4.1$.

We can summarize these two solutions in two tables.

Case 1

a: 28.5	A: 65°
b: 30.0	B: 72.6°
c: 21.2	C: 42.4°

Case 2

a: 28.5	A: 65°
b: 30.0	B: 107.4°
c: 4.1	C: 7.6°

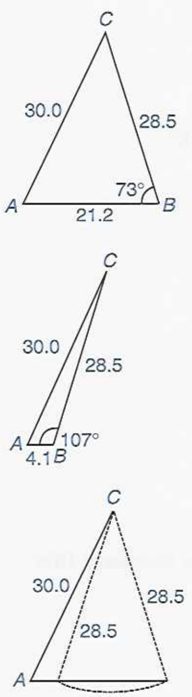


Figure 6–2

Figure 6–2 shows the two triangles. The last figure shows both triangles together, where we can see why the ambiguous case was possible—with the given information ($a = 28.5$, $b = 30.0$, $A = 65^\circ$), side a could be in one of two positions, giving two possible triangles.

As a final check on our work, we observe that the sum of the angles in each case is 180°, and that in each case the longest side is opposite the largest angle and the shortest side is opposite the smallest angle. These are facts that should be true for any triangle.

2. $b = 51.2, c = 32.1, B = 6.1°$

a: ?	A: ?	Make a table of values
b: 51.2	B: 6.1°	
c: 32.1	C: ?	

$$\frac{\sin A}{a} = \frac{\sin 6.1°}{51.2} = \frac{\sin C}{32.1}$$ Fill values into the law of sines

$$\sin C = \frac{32.1 \sin 6.1°}{51.2}$$ Using the last two ratios

$C' \approx 3.82°$, which we will round to 3.8° in the final answer.
$C \approx 3.82°$ or $180° - 3.82° \approx 176.18°$.

Case 1: $C \approx 3.82°$

a: ?	A: ?
b: 51.2	B: 6.1°
c: 32.1	C: 3.82°

$$A \approx 180° - 3.82° - 6.1° = 170.08°$$

$$\frac{\sin 170.08°}{a} = \frac{\sin 6.1°}{51.2}$$ Use the first two ratios

$$a = \frac{51.2 \sin 170.08°}{\sin 6.1°} \approx 83.00$$

This finishes case 1: $A \approx 170.08°$, $a \approx 83.0$.

Case 2: $C \approx 176.18°$

a: ?	A: ?
b: 51.2	B: 6.1°
c: 32.1	C: 176.18°

$A \approx 180° - 176.18° - 6.1° = -2.28°$
Since an angle in a triangle cannot have negative measure this case does not produce a solution.

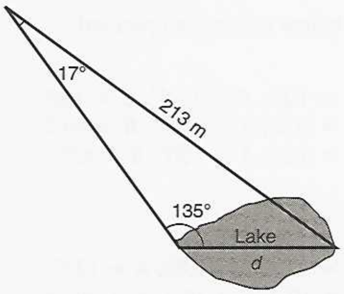
Thus, the solution to case 1 is the only solution.

a: 83.0	A: 170.1°
b: 51.2	B: 6.1°
c: 32.1	C: 3.8°

The law of sines is used in many applications of trigonometry.

■ Example 6–1 C

Solve the following problems using the law of sines.



1. A surveyor made the measurements shown in the figure to measure the distance d across a lake. Find the distance to the nearest meter.

Using the law of sines, we know that

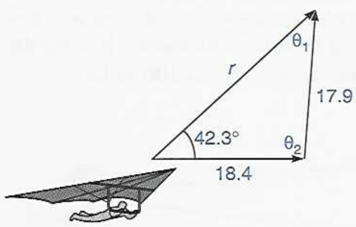
$$\frac{\sin 135°}{213} = \frac{\sin 17°}{d}$$

$$\frac{213d}{1} \cdot \frac{\sin 135°}{213} = \frac{213d}{1} \cdot \frac{\sin 17°}{d}$$

$$d \sin 135° = 213 \sin 17°$$

$$d = \frac{213 \sin 17°}{\sin 135°} \approx 88.1 \text{ meters}$$

Thus, to the nearest meter the distance across the lake is 88 meters.



2. The figure illustrates the following situation. A hang glider is flying at 18.4 mph pointed due east, with the wind blowing at 17.9 mph in the direction shown. The result is that the hang glider travels in a direction 42.3° north of east, with a ground speed of r mph. Assuming that θ_1 is acute, find the ground speed r and the direction of the wind, θ_2.

Using the law of sines, we know

$$\frac{\sin 42.3°}{17.9} = \frac{\sin \theta_1}{18.4} = \frac{\sin \theta_2}{r}$$

$$\sin \theta_1 = \frac{18.4 \sin 42.3°}{17.9}$$

$$\theta_1' \approx 43.77°$$

$$\theta_1 = \theta_1' \approx 43.77° \qquad \theta_1 \text{ is an acute angle}$$

$$\theta_2 \approx 180° - 42.3° - 43.77° = 93.93°$$

We can now find r.

$$\frac{\sin 42.3°}{17.9} = \frac{\sin 93.93°}{r}$$

$$r = \frac{17.9 \sin 93.93°}{\sin 42.3°} \approx 26.5$$

Thus, the hang glider is traveling with a ground speed of 26.5 mph, and the wind is blowing in the direction 93.9° north of west (or 3.9° east of north). ■

Mastery points

Can you

- State and use the law of sines to solve oblique triangles?
- Recognize and solve the ambiguous case when using the law of sines?

Exercise 6–1

In the following problems round answers to the same number of decimal places as the data, unless otherwise specified.

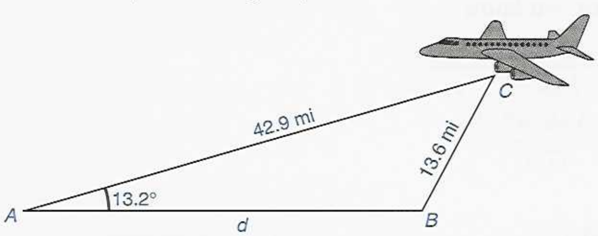
Solve the following oblique triangles using the law of sines.

1. $a = 12.5, A = 35°, B = 49°$
2. $b = 17.1, B = 100°, C = 10°$
3. $a = 1.25, B = 13.6°, C = 132°$
4. $c = 9.04, A = 51.6°, B = 40.0°$
5. $b = 92.5, A = 47°, B = 100°$
6. $c = 10.2, A = 16.7°, B = 89.2°$
7. $a = 0.452, A = 67.6°, C = 91.8°$
8. $b = 0.508, B = 13.1°, C = 5.2°$
9. $c = 5.00, A = 100°, B = 45°$
10. $a = 10.9, B = 76.9°, C = 100°$

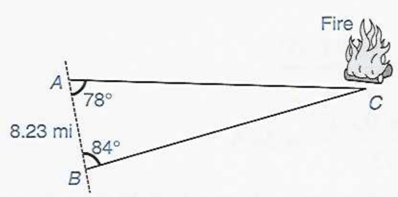
Solve the following oblique triangles using the law of sines.

11. $a = 12.5, b = 13.2, B = 49°$
12. $b = 37.1, c = 21.3, B = 100°$
13. $a = 4.25, c = 2.86, A = 132°$
14. $c = 9.04, a = 21.3, C = 10.0°$
15. $b = 92.5, c = 98.6, B = 43.7°$
16. $c = 10.2, a = 16.7, A = 89.2°$
17. $a = 4, b = 22, A = 30°$
18. $a = 0.452, c = 0.606, C = 91.8°$
19. $b = 6.35, c = 4.29, C = 42.3°$
20. $b = 0.508, c = 1.09, C = 5.2°$
21. $c = 5.00, b = 8.00, B = 45.0°$
22. $a = 10.9, c = 16.9, C = 100.0°$

23. Ground-based radar at point A determines that the angle of elevation to an aircraft 42.9 miles away is 13.2°. Radar at point B is on a straight line between a point on the ground directly below the aircraft and the radar at A and determines that the same aircraft is 13.6 miles away from point B. To the nearest 0.1 mile, find the distance from A to B. (See the diagram.)

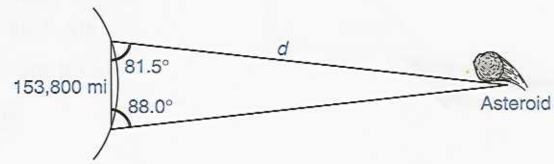


24. Two forest rangers sight a fire. Their reports are plotted on a map and yield the results shown in the diagram.

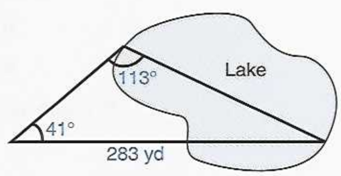


If the locations of the rangers are 8.23 miles apart, how far is the fire from Station A, to the nearest 0.1 mile?

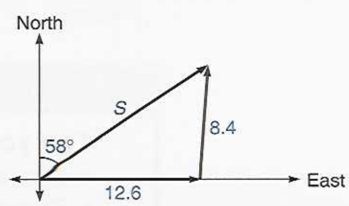
25. The diagram shows a situation in which astronomers made measurements at two locations of a new, slow-moving asteroid. Using their measurements, find the distance d to the asteroid, to the nearest 100 miles.



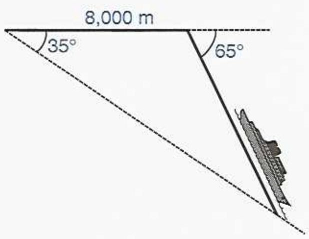
26. A surveyor made the measurements shown in the diagram. Find the distance across the lake, to the nearest foot.



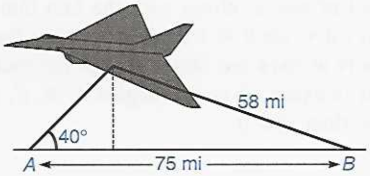
27. The diagram illustrates a situation in which a ship is moving at 12.6 knots heading due east. It is moving through a current moving east of north at 8.4 knots. The result is that the ship is moving at an angle of 58° east of north. Find the true speed S of the ship.



28. A ship travels due east to a point 8,000 meters from its starting point. It then turns toward the south through an angle of 65° and proceeds until it crosses a line of sight from the starting position to itself that is 35° south of east. At this point, how far is the ship from its starting point, to the nearest 10 meters?

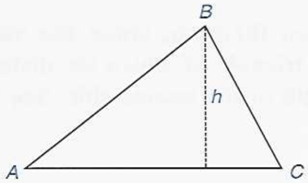


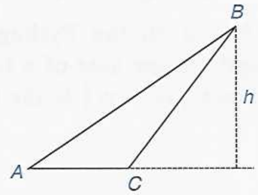
29. Two cities are 75 miles apart. An aircraft that is between the two cities is being tracked from radar in each city. City A's radar shows that the aircraft is at an angle of elevation of 40°; city B's radar shows that the slant distance of the plane to city B is 58 miles and that the angle of elevation there is less than 40°. What is the slant distance d from the plane to city A, to the nearest mile?



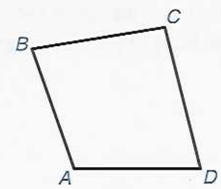
30. Find the height of the aircraft in problem 29, to the nearest 100 feet (1 mile = 5,280 feet). Use the diagram for help.

31. Show that in any oblique triangle ABC, if h is the altitude of the triangle relative to side b, then an expression for h is $h = \dfrac{b \cdot \tan A \cdot \tan C}{\tan A + \tan C}$. Use the diagrams for help.

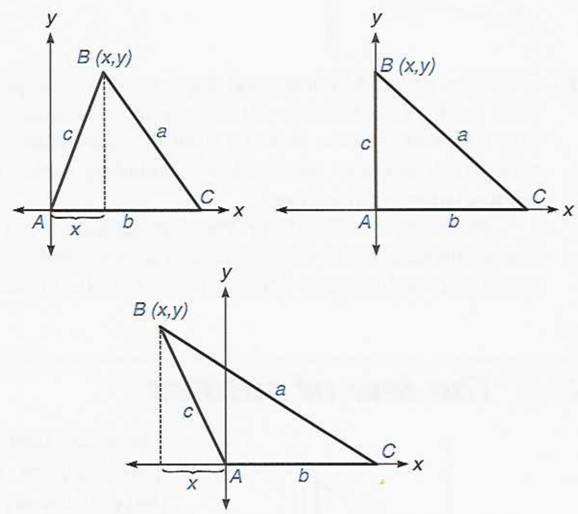




32. Quadrilateral $ABCD$ is shown in the diagram. $AB = 17.3$, $AD = 18.9$, angle $A = 110°$, angle $ABD = 52°$, angle $BDC = 41°$, angle $C = 93°$. Find the length of CD to the nearest 0.1. (*Hint:* Draw diagonal BD.)



33. Show that in any triangle ABC, (1) $a = b \cos C + c \cos B$, (2) $b = c \cos A + a \cos C$, and (3) $c = a \cos B + b \cos A$. (*Hint:* The diagram shows angle A in standard position when the angle is acute, right, or obtuse.)



Show why the statements

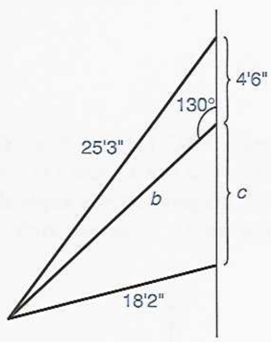
$$\cos A = \frac{x}{c} \text{ and } \cos C = \frac{b - x}{a}$$

are true in each case, and then use them to show that (2) is true.

34. An Army observation point is 325 yards northeast (i.e., 45° north of east) of a second point. At this point a tank is sighted on a line of sight 37° south of east. The same tank is sighted at the second point along a line of sight 18° north of east. To the nearest yard, how far is the tank from the first observation point?

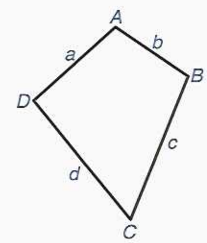
35. Recall that the formula for the area of a triangle is $\frac{1}{2}bh$ (one half the product of the base and height). Use the formula to show that the area of any triangle ABC is $\frac{1}{2}bc \sin A$.

36. The figure shows three wires that are attached from a common point to the side of a building. Find the lengths of b and c, to the nearest inch.



37. The Rhind Mathematical Papyrus is an Egyptian work on mathematics. It dates to the sixteenth century B.C., and contains material from the nineteenth century B.C. It contains 84 problems, including tables for manipulations of fractions.

Problems 51–53 of the Papyrus include the following formula for finding the area of a four-sided figure like $ABCD$ in the figure: $\frac{1}{2}(a + c) \times \frac{1}{2}(b + d)$. (Observe that $\frac{1}{2}(a + c)$ is the average length of the two sides a and c; the same is true for $\frac{1}{2}(b + d)$.) This formula is inaccurate. Except for rectangles, it gives an answer that is too large. It was used for the purpose of taxing land, which shows that there is not always an economic incentive to get the correct answer.



Problem 35 shows that the area of any triangle ABC is $\frac{1}{2}bc \sin A$. It is also $\frac{1}{2}ac \sin B$, or $\frac{1}{2}ab \sin C$. Geometrically, this is one-half the product of two sides and the sine of the angle between those two sides.

a. Use this result to show that the area of the four-sided figure can be described by

$$\frac{1}{4}(ab \sin A + ad \sin D + bc \sin B + cd \sin C)$$

b. Use the result of part a, along with the fact that for $0 < \theta < 180°$, $0 < \sin \theta < 1$ to show that the Egyptian formula is always too large, except for rectangles, when it is exact. (Assume angles A, B, C, and D are all less than $180°$.)

6–2 The law of cosines

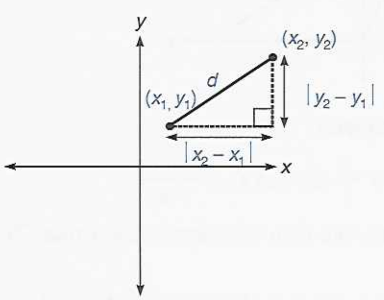


Figure 6–3

There are some oblique triangles that cannot be solved using the law of sines. This happens when we do not know any of the three ratios completely. In these cases we use the *law of cosines.* To develop this law, we will use the **distance formula** of analytic geometry: If (x_1,y_1) and (x_2,y_2) are two points in the x-y coordinate plane, then the distance d between them is defined as

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This definition is based on the Pythagorean theorem, since the value $|x_2 - x_1|$ is the length of one side of a right triangle of which the distance d is the hypotenuse, and $|y_2 - y_1|$ is the length of the second side. See figure 6–3.

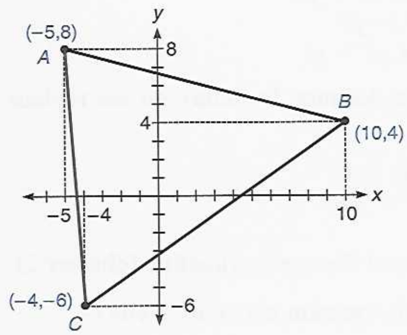
■ ***Example 6–2 A***

1. Find the distance between the points $(-4,7)$ and $(2,10)$.

   Using $(x_1,y_1) = (-4,7)$ and $(x_2,y_2) = (2,10)$, we have

$$\begin{aligned} d &= \sqrt{[2-(-4)]^2 + (10-7)^2} \\ &= \sqrt{36+9} \\ &= \sqrt{45} \\ &= \sqrt{9\cdot 5} \\ &= 3\sqrt{5} \end{aligned}$$

Note The points could have been used in the reverse order with the same result. That is, $(x_1,y_1) = (2,10)$ and $(x_2,y_2) = (-4,7)$.



2. Assume that an optics table is coordinatized in the usual rectangular coordinate system. Three mirrors are located on the table at points $A\ (-5,8)$, $B\ (10,4)$, and $C\ (-4,-6)$. Find the distance traversed by a laser beam traveling from A to B to C and back to A, both exactly and to the nearest 0.1. See the figure.

   We use the distance formula: $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$. To find the distance from A to B, we can let A be the first point and B be the second. Then the formula becomes

$$AB = \sqrt{(x_B-x_A)^2 + (y_B-y_A)^2}.$$

We fill in the given values.

$$\begin{aligned} AB &= \sqrt{[10-(-5)]^2 + (4-8)^2} \\ &= \sqrt{15^2 + (-4)^2} \\ &= \sqrt{241} \approx 15.52 \text{ (to two decimal places)} \end{aligned}$$

To find the distance from A to C, we can let A be the first point and C be the second.

$$\begin{aligned} AC &= \sqrt{(x_C-x_A)^2 + (y_C-y_A)^2} \\ &= \sqrt{[(-4)-(-5)]^2 + [(-6)-8]^2} \\ &= \sqrt{1^2 + (-14)^2} \\ &= \sqrt{197} \approx 14.04 \end{aligned}$$

To find the distance from B to C, we can let B be the first point and C be the second.

$$\begin{aligned} BC &= \sqrt{(x_C-x_B)^2 + (y_C-y_B)^2} \\ &= \sqrt{[(-4)-10]^2 + [(-6)-4]^2} \\ &= \sqrt{(-14)^2 + (-10)^2} \\ &= \sqrt{296} = \sqrt{4(74)} = 2\sqrt{74} \approx 17.20 \end{aligned}$$

The exact total distance is

$$\begin{aligned} AB + AC + BC &= \sqrt{241} + \sqrt{197} + 2\sqrt{74} \text{ (exactly)} \\ &\approx 46.8 \end{aligned}$$

■

We now use the distance formula in our development of the law of cosines.

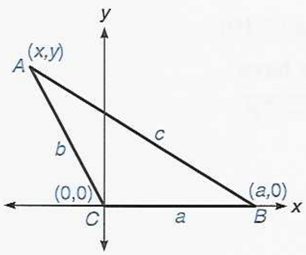


Figure 6–4

Let ΔABC be any triangle. Put angle C in standard position, and call (x,y) the point at vertex A, as illustrated in figure 6–4.

Note The figure shows C as an obtuse angle. The algebraic statements that follow do not use this fact, however. Thus, they would also apply if C were acute or right.

Now apply the distance formula to distance c.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Use $(x_2,y_2) = (a,0)$ and $(x_1,y_1) = (x,y)$.

$$\begin{aligned} c &= \sqrt{(a - x)^2 + (0 - y)^2} \\ c^2 &= (a - x)^2 + (-y)^2 \\ &= a^2 - 2ax + x^2 + y^2 \end{aligned}$$

We know that $x^2 + y^2 = b^2$ (also by the distance formula), so we replace $x^2 + y^2$ in the above equation by b^2.

$$\begin{aligned} c^2 &= a^2 - 2ax + b^2 \\ &= a^2 + b^2 - 2ax \end{aligned}$$

We know that $\cos C = \dfrac{x}{b}$ by the definition of the cosine function (chapter 2). Thus, $x = b \cos C$, and we replace x in the equation above by $b \cos C$.

$$c^2 = a^2 + b^2 - 2ab \cos C$$

The equation $c^2 = a^2 + b^2 - 2ab \cos C$ is called the **law of cosines** for angle C. If we had put angle A or angle B in standard position, we would have arrived at two other versions of this law. All three versions are:

The law of cosines

Law of cosines for angle A: $a^2 = b^2 + c^2 - 2bc \cos A$

Law of cosines for angle B: $b^2 = a^2 + c^2 - 2ac \cos B$

Law of cosines for angle C: $c^2 = a^2 + b^2 - 2ab \cos C$

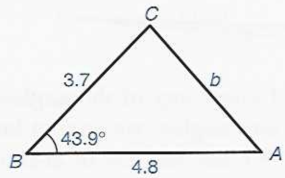
Concept

The law of cosines states that in any triangle the square of the length of one side equals the sum of the squares of the lengths of the other two sides less twice the product of these lengths and the cosine of the angle opposite the first side.

When solving oblique triangles, the law of cosines should be used whenever the law of sines cannot be used.

■ ***Example 6–2 B***

1. Solve oblique triangle ABC if $a = 3.7$, $c = 4.8$, and angle $B = 43.9°$. See the figure.



Observe that we do not know any of the three ratios of the law of sines completely: we know the length of side a but not the measure of angle A, the length of side c but not the measure of angle C, and the measure of angle B but not the length of side b. This indicates that we must use the law of cosines to help solve the problem.

Since we know angle B, we use the form of the law of cosines in which angle B appears:

$$\begin{aligned} b^2 &= a^2 + c^2 - 2ac \cos B \\ &= 3.7^2 + 4.8^2 - 2(3.7)(4.8)(\cos 43.9°) \\ b &= \sqrt{3.7^2 + 4.8^2 - 2(3.7)(4.8)(\cos 43.9°)} \qquad \boxed{\text{CS 1}} \\ &\approx 3.337068251\ldots \end{aligned}$$

(which we will round off to 3.337 for now and to 3.3 in the final answer).

Now we use the law of sines to find one of the angles, either A or C. It is best to find angle A first, because we know angle A is not the largest angle in the triangle (angle C is) and, therefore, angle A must be acute. This will eliminate the ambiguous case.

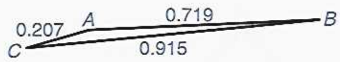
$$\begin{aligned} \frac{3.7}{\sin A} &\approx \frac{3.337}{\sin 43.9°} \\ 3.337(\sin A) &\approx 3.7(\sin 43.9°) \\ \sin A &\approx \frac{3.7(\sin 43.9°)}{3.337} \\ A' &\approx 50.2° \end{aligned}$$

(remember, A' is the reference angle for angle A). We know that A is acute, so we do not have to worry about the supplement of A (which we normally do whenever we find an angle using the law of sines), so $A \approx 50.2°$. Also, $C = 180° - A - B \approx 180° - 50.2° - 43.9° \approx 85.9°$. Since we now know all three angles and all three sides of the triangle, we are done.

$$A \approx 50.2°,\ B = 43.9°,\ C \approx 85.9°$$
$$a = 3.7,\ b = 3.3,\ c = 4.8$$

Note As illustrated in the last example, it is never necessary to use the law of cosines more than once to solve a triangle. We complete the solution with the law of sines.

2. Solve oblique triangle ABC if $a = 0.915$, $b = 0.207$, and $c = 0.719$. See the figure.



We do not know any of the angles, so we cannot use the law of sines. (Without any angles, we cannot know any of the three ratios in the law of sines.) We use the law of cosines to find one of the angles.

It is best to find the largest angle first; this will be angle A since it is opposite the longest side a. The reason for finding the largest angle first is explained in the note below. We must use the form of the law of cosines that includes angle A.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

We often solve for $\cos A$ before we use the law of cosines.

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cos A \\ 2bc \cos A &= b^2 + c^2 - a^2 \\ \cos A &= \frac{b^2 + c^2 - a^2}{2bc} \end{aligned}$$

Substituting values, we get

$$\begin{aligned} \cos A &= \frac{0.207^2 + 0.719^2 - 0.915^2}{2(0.207)(0.719)} \\ &\approx -0.9320. \\ A &\approx 158.74^\circ \end{aligned}$$

CS 2

Now we know the measure of the largest angle: $A \approx 158.74^\circ$. We can now use the law of sines to find another angle. We must use the ratio $\dfrac{a}{\sin A}$ since A is the only angle measure we know. To find angle B we use

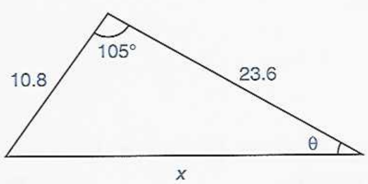
$$\frac{0.915}{\sin 158.74^\circ} \approx \frac{0.207}{\sin B}$$

to find that

$$B' \approx 4.70^\circ.$$

Since we have already found the largest angle A, we know that the remaining two angles are acute, and so we do not have to worry about the supplement of B'; thus, $B \approx 4.70^\circ$ and $C \approx 180^\circ - 158.7^\circ - 4.7^\circ = 16.6^\circ$. The triangle is solved because we now know all three sides (which were given) and all three angles. $A \approx 158.7^\circ$, $B \approx 4.7^\circ$, $C \approx 16.6^\circ$.

Note There is no ambiguous case for the law of cosines because the range of the inverse cosine function includes all angles from 0° to 180°. If an angle is not acute, its cosine will be negative (as above), and then we know the angle is between 90° and 180°. Thus, *when finding an angle of a triangle using the law of cosines* (as above), *it is best to find the largest angle first.* If the largest angle is obtuse we will find this out directly; either way, the remaining angles are acute and, therefore, easy to find with the law of sines.



3. A sheet metal worker must make the triangular pattern shown in the figure. Find the angle of inclination θ and the length of side x.

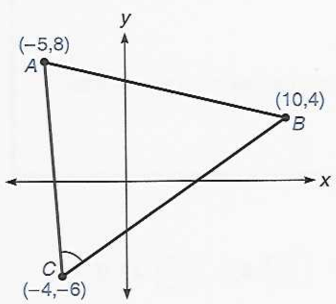
Using the law of cosines, we see that

$$\begin{aligned} x^2 &= 10.8^2 + 23.6^2 - 2(10.8)(23.6)\cos 105° \\ &\approx 805.54 \\ x &\approx 28.38 \end{aligned}$$

We can now use the law of sines to find θ.

$$\frac{28.38}{\sin 105°} \approx \frac{10.8}{\sin\theta}$$
$$28.38 \sin\theta \approx 10.8 \sin 105°$$
$$\sin\theta \approx \frac{10.8 \sin 105°}{28.38} \approx 0.3676$$
$$\theta' \approx 21.6°$$

θ is acute because it is not the largest angle in the triangle, so $\theta = 21.6°$.



4. Find the measure of angle ACB in part 2 of example 6–2 A to the nearest 0.1°. Recall that we have a triangle whose vertices were the points A $(-5,8)$, B $(10,4)$, and C $(-4,-6)$. See the figure.

In part 2 of example 6–2 A we used the distance formula to establish

$$\begin{aligned} AB &= \sqrt{241} \approx 15.52 \\ AC &= \sqrt{197} \approx 14.04 \\ BC &= 2\sqrt{74} \approx 17.20 \end{aligned}$$

To find angle ACB, we use the law of cosines with angle C.

$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab\cos C \\ \cos C &= \frac{a^2 + b^2 - c^2}{2ab} \\ &= \frac{(2\sqrt{74})^2 + (\sqrt{197})^2 - (\sqrt{241})^2}{2(2\sqrt{74})(\sqrt{197})} \\ &= \frac{296 + 197 - 241}{4(\sqrt{74})(\sqrt{197})} \approx 0.5218 \end{aligned}$$

$C \approx 58.5°$ $\boxed{\text{CS 3}}$

■ ***Example 6–2 C***

In triangle ABC, $a = 4$, $b = 6$, $c = 12$. Solve the triangle.

We must use the law of cosines to solve this triangle. If we wish to find the largest angle, which is C, we use

$$\begin{aligned}\cos C &= \frac{a^2 + b^2 - c^2}{2ab} \\ &= \frac{4^2 + 6^2 - 12^2}{2(4)(6)} \\ &= \frac{-92}{48} \\ &\approx -1.9167\end{aligned}$$

Note that this result shows that there is no angle C, since $|\cos x| \leq 1$ for any value of x. Thus, there is no solution to this triangle.

Example 6–2 C illustrates a situation in which the law of cosines shows us that there is no solution. There is no solution because $a + b < c$. It is a fact that any two sides of a triangle must have a total length greater than the third side. This is often called the **triangle inequality.** Any time we are given the lengths of three sides of a triangle, it is a good idea to check that the sum of the lengths of any two sides is greater than the length of the remaining side.

Mastery points

Can you

- State the three forms of the law of cosines?
- Use the law of cosines to solve triangles when the law of sines cannot be used?
- Use the distance formula?

Calculator steps

1. Ⓐ 3.7 $\boxed{x^2}$ $\boxed{+}$ 4.8 $\boxed{x^2}$ $\boxed{-}$ 2 $\boxed{\times}$ 3.7 $\boxed{\times}$ 4.8 $\boxed{\times}$ 43.9 $\boxed{\text{COS}}$ $\boxed{=}$ $\boxed{\sqrt{x}}$

Ⓟ 3.7 $\boxed{x^2}$ 4.8 $\boxed{x^2}$ $\boxed{+}$ 2 $\boxed{\text{ENTER}}$ 3.7 $\boxed{\times}$ 4.8 $\boxed{\times}$ 43.9 $\boxed{\text{COS}}$ $\boxed{\times}$ $\boxed{-}$ $\boxed{\sqrt{x}}$

$\boxed{\text{TI-81}}$ $\boxed{\sqrt{\ }}$ $\boxed{(}$ 3.7 $\boxed{x^2}$ $\boxed{+}$ 4.8 $\boxed{x^2}$ $\boxed{-}$ 2 $\boxed{\times}$ 3.7 $\boxed{\times}$ 4.8 $\boxed{\text{COS}}$ 43.9 $\boxed{)}$ $\boxed{\text{ENTER}}$

2. Ⓐ .207 $\boxed{x^2}$ $\boxed{+}$ .719 $\boxed{x^2}$ $\boxed{-}$ .915 $\boxed{x^2}$ $\boxed{=}$ $\boxed{\div}$ 2 $\boxed{\div}$ .207 $\boxed{\div}$ .719 $\boxed{=}$ $\boxed{\text{INV}}$ $\boxed{\text{COS}}$

Ⓟ .207 $\boxed{x^2}$ .719 $\boxed{x^2}$ $\boxed{+}$ .915 $\boxed{x^2}$ $\boxed{-}$ 2 $\boxed{\div}$ .207 $\boxed{\div}$ .719 $\boxed{\div}$ $\boxed{\text{COS}^{-1}}$

$\boxed{\text{TI-81}}$ $\boxed{\cos^{-1}}$ $\boxed{(}$ $\boxed{(}$ .207 $\boxed{x^2}$ $\boxed{+}$ .719 $\boxed{x^2}$ $\boxed{-}$ .915 $\boxed{x^2}$ $\boxed{)}$ $\boxed{\div}$ $\boxed{(}$ 2 $\boxed{\times}$ .207 $\boxed{\times}$ .719 $\boxed{)}$ $\boxed{)}$ $\boxed{\text{ENTER}}$

3. Ⓐ 296 $\boxed{+}$ 197 $\boxed{-}$ 241 $\boxed{=}$ $\boxed{\div}$ 4 $\boxed{\div}$ 74 $\boxed{\sqrt{x}}$ $\boxed{\div}$ 197 $\boxed{\sqrt{x}}$ $\boxed{=}$ $\boxed{\text{INV}}$ $\boxed{\text{COS}}$

Ⓟ 296 $\boxed{\text{ENTER}}$ 197 $\boxed{+}$ 241 $\boxed{-}$ 4 $\boxed{\div}$ 74 $\boxed{\sqrt{x}}$ $\boxed{\div}$ 197 $\boxed{\sqrt{x}}$ $\boxed{\div}$ $\boxed{\text{COS}^{-1}}$

$\boxed{\text{TI-81}}$ $\boxed{\cos^{-1}}$ $\boxed{(}$ $\boxed{(}$ 296 $\boxed{+}$ 197 $\boxed{-}$ 241 $\boxed{)}$ $\boxed{\div}$ $\boxed{(}$ 4 $\boxed{\sqrt{\ }}$ 74 $\boxed{\sqrt{\ }}$ 197 $\boxed{)}$ $\boxed{)}$ $\boxed{\text{ENTER}}$

Exercise 6–2

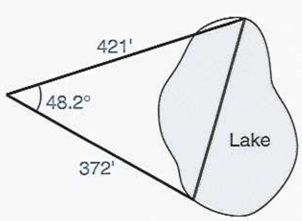
Find the distance between the following points.

1. (3,−4), (−2,6)
2. (3,−1), (4,−8)
3. $\boxed{3.}$ (−5,−2), (10,−1)
4. (−2,−3), (−4,−1)
5. (0,−3), (−9,−12)
6. (10,−1), (2,0)

Solve the following oblique triangles. You will have to use the law of cosines as the first step. Round your answers to the same number of decimal places as the data.

7. $a = 3.2$, $b = 5.9$, $C = 39.4°$
8. $a = 4.9$, $b = 3.2$, $C = 78.2°$
9. $\boxed{9.}$ $b = 61.3$, $c = 23.9$, $A = 124.0°$
10. $b = 123.0$, $c = 89.4$, $A = 19.5°$
11. $a = 31.4$, $c = 17.0$, $B = 100.3°$
12. $a = 67.25$, $c = 13.56$, $B = 76.30°$
13. $\boxed{13.}$ $a = 23.5$, $b = 19.4$, $c = 35.0$
14. $a = 61.7$, $b = 80.0$, $c = 102.0$
15. $a = 0.214$, $b = 0.399$, $c = 0.500$
16. $a = 1.03$, $b = 0.98$, $c = 1.75$
17. $a = 13.2$, $b = 5.9$, $C = 139.4°$
18. $a = 14.9$, $b = 13.2$, $C = 45.0°$
19. $b = 61.3$, $c = 43.9$, $A = 24.5°$
20. $b = 23.9$, $c = 89.4$, $A = 79.5°$
21. $a = 30.0$, $c = 20.0$, $B = 112.0°$
22. $a = 6.72$, $c = 1.55$, $B = 76.35°$
23. $a = 235$, $b = 194$, $c = 354$
24. $a = 61.7$, $b = 80.0$, $c = 42.0$
25. $a = 0.21$, $b = 0.49$, $C = 1.50°$
26. $a = 10.0$, $b = 13.9$, $c = 17.5$

27. A surveyor made the measurements shown in the diagram to calculate the distance across a lake.



Compute the distance to the nearest 0.1 foot.

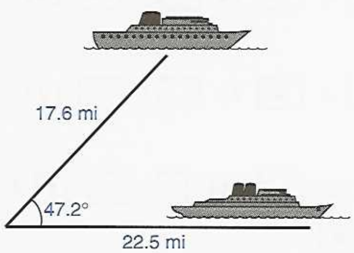
28. Calculate the measure of the smallest angle in a triangle whose sides have measure 22.1 cm, 32.6 cm, and 40.5 cm.

29. Calculate the measure of the largest angle in a triangle whose sides have measure 12.3 in., 16.2 in., and 19.0 in.

30. A numerically controlled laser cloth cutter is being set up to cut a triangular pattern. The vertices of the triangle are at A (2,5), B (4,8), and C (5,12).
    a. Find the length of side AB.
    b. Determine the measure of the three angles to the nearest 0.1°.

31. In the same situation as in problem 30, the vertices of another piece of triangular cloth are determined to be at $A(0,5)$, $B(2,3)$, and $C(8,4)$. Determine the measure of the largest of the three angles A, B, or C, to the nearest 0.1°.

32. Two ships are being tracked by radar. One ship is determined to be 17.6 miles from the radar, while the second is 22.5 miles from the radar. The lines of sight from the radar to the two ships form an angle of 47.2°. (See the diagram.) Find the distance between the two ships to the nearest 0.1 mile.



33. In the situation described in problem 32, what would be the angle formed by the two lines of sight to the ships if the ships were 31.5 miles apart?

34. A ship leaves a harbor heading due east and travels 17.3 km. It then turns north through a 33° angle and travels for another 22.0 km. How far is the ship from its starting point, to the nearest kilometer?

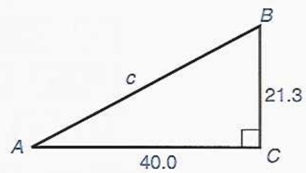
35. A plane takes off and travels southeast (45° south of east) for 27 miles, then turns due south and travels for 16 miles. How far is it from its starting position, to the nearest mile?

36. The points (5,3), (−2,1), and (1,−4) form a triangle. Find the measure of the smallest angle in this triangle, to the nearest 0.1°.

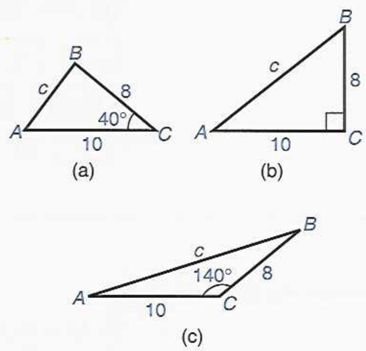
37. Find the measure of the largest angle in the triangle of problem 36, to the nearest 0.1°.

38. Two observers are 87 meters apart, and a range finder shows that a certain building is 111 meters from one observer and 114 meters from the other. What is the angle formed by the two lines of sight from the building to the observers, to the nearest degree?

39. The triangle in the diagram is a right triangle. Can the law of cosines be used to find the length of c? Compare using the law of cosines to using the Pythagorean theorem.



40. The diagram shows three triangles in which angle C is (a) acute, (b) right, and (c) obtuse.



If we use the Pythagorean theorem and apply it to the right triangle in (b) and apply the law of cosines to angle C in all three situations, the resulting equations are

$$c^2 = a^2 + b^2 \text{ and}$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

We can view the expression $-2ab\cos C$ as a ''correction factor'' to the Pythagorean theorem that will give the correct result, even when angle C is not 90°.

Solve for side c in each case, using the law of cosines, and discuss how the correction factor ''knows'' when to increase the length of side c (as in [c]) and when to decrease it (as in [a]).

6–3 Vectors

If we know that a plane flying over a certain spot is flying at 100 mph, we cannot tell where it will be in 1 hour without also knowing its direction. This combination of speed and direction is called *velocity*. Many other natural phenomena are described by a magnitude and direction: forces of all types, accelerations, alternating voltage in electricity theory are all examples. A conceptual tool used to describe two such pieces of information is the **vector.** It is noteworthy that the concept of vectors extends into an area of mathematics called *linear algebra,* which has applications in every field of knowledge, from physics and economics to medicine and sociology.

We imagine a vector as a directed line segment. That is, a finite portion of a straight line that is considered to be pointing in one direction. Our representation of vectors would commonly be called "arrows." Examples of vectors are shown in figure 6–5. Observe that we use the terms **head** and **tail** to describe the "end" and "beginning" of a vector, respectively, and that we often use capital letters, such as A and B, to name a vector.

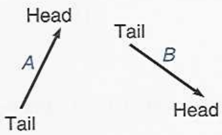


Figure 6–5

One way to describe a vector is by specifying its length and direction. The length is called the **magnitude** of the vector, and for a vector A we denote its magnitude by $|A|$ (the same notation as that for absolute value of a real number). The **direction** of a vector is specified by an angle, θ, usually specified in degrees, measured in the same way as angles in standard position. When we specify a vector this way we say it is in **polar form.**

> **Polar form of a vector**
> A vector A in polar form is the ordered pair $A = (|A|, \theta_A)$
> $|A|$ is the magnitude of vector A; $|A| \geq 0$.
> θ_A is the direction of vector A.

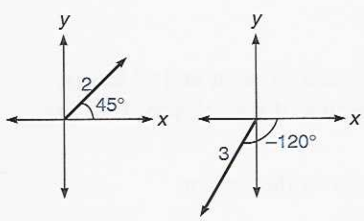


Figure 6–6

Note The polar form of a vector is not unique; all coterminal values of θ_A are equivalent.

Figure 6–6 illustrates the vectors $(2,45°)$ and $(3,-120°)$.

A vector can also be specified by using its *relative rectangular coordinates.* For a vector in standard position this is equivalent to specifying the coordinates of its head (see figure 6–8). If a vector is not in standard position, its relative rectangular coordinates are relative to the coordinates of the vector's tail. We refer to these coordinates as describing the **rectangular form** of a vector.

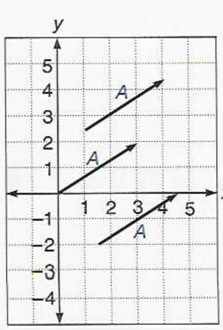


Figure 6–7

> **Rectangular form of a vector**
> A vector A in rectangular form is $A = (A_x, A_y)$, where A_x is called the **horizontal component** of the vector and A_y is called the **vertical component** of the vector.

Figure 6–7 illustrates the vector $A(3,2)$, shown in three different positions.

Converting polar form to rectangular form

There is a useful relation for converting the polar form of a vector to its rectangular form. If we recall the definitions of section 2–2 for an angle in standard position, we can see the following.

> **To convert polar form to rectangular form**
> Given vector $A = (|A|,\theta_A) = (A_x, A_y)$,
> $$A_x = |A| \cos \theta_A \quad \text{and} \quad A_y = |A| \sin \theta_A$$

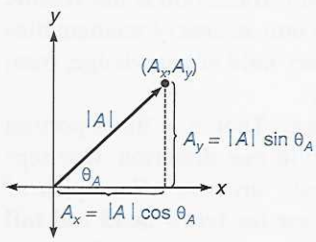


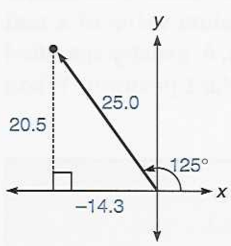
Figure 6–8

This is true because, by the definitions of section 5–2, $\cos \theta_A = \dfrac{A_x}{|A|}$ and $\sin \theta_A = \dfrac{A_y}{|A|}$. Figure 6–8 illustrates this relationship.

Example 6–3 A illustrates converting a vector in polar to form to its rectangular form.

■ Example 6–3 A

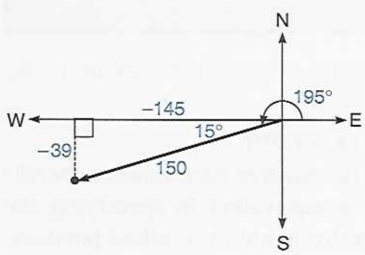
Convert from polar to rectangular form.



1. Convert the polar form of the vector to the rectangular form (approximate to the nearest tenth). $A = (25.0,125°)$.

$$A_x = |A| \cos \theta_A = 25.0 \cos 125° \approx -14.3$$
$$A_y = |A| \sin \theta_A = 25.0 \sin 125° \approx 20.5$$

Thus, the rectangular form is $(-14.3,20.5)$ (see the figure).



2. An aircraft is moving in the direction 15° south of west at 150 knots. Find the east-west and north-south components of its velocity V to the nearest knot and interpret the results.

As we see in the figure the aircraft's velocity is the vector $V = (150,195°)$. The east-west component is

$$\begin{aligned} V_x &= |V| \cos \theta_V \\ &= 150 \cos 195° \approx -145 \end{aligned}$$

The north-south component is

$$\begin{aligned} V_y &= |V| \sin \theta_V \\ &= 150 \sin 195° \approx -39 \end{aligned}$$

Thus, the aircraft is moving west at 145 knots and south at 39 knots. ■

Converting rectangular form to polar form

When we convert from rectangular to polar form we always give the direction of the vector θ_V so that it has the smallest possible absolute value. This means *we will choose* θ_V *so that* $-180° < \theta_V \leq 180°$. One reason for doing this is that this is the result obtained from electronic calculators (see the discussion following example 6–3 B).

Examining figure 6–8 shows that, for a given vector $V = (V_x, V_y) = (|V|, \theta_V)$, $|V| = \sqrt{V_x^2 + V_y^2}$, and $\tan\theta'_V = \dfrac{V_y}{V_x}$ if $V_x \neq 0$. The angle $\theta'_V = \tan^{-1}\dfrac{V_y}{V_x}$ is only the reference angle and θ_V depends on the quadrant in which the vector occurs.

Reference angles obtained with the inverse tangent function fall in the range $-90° < \theta' < 90°$ (quadrant I and quadrant IV). If $V_x > 0$ this is the value of θ_V, since that angle should be in quadrant I or quadrant IV.

If $V_x < 0$, θ_V can be obtained by adding or subtracting 180° to or from θ'_V. If $\theta'_V < 0$, add 180°, and if $\theta'_V > 0$, subtract 180°. This can be incorporated into a rule as follows.

To convert rectangular form to polar form

Given vector $V = (V_x, V_y) = (|V|, \theta_V)$. Then,

$$|V| = \sqrt{V_x^2 + V_y^2},\ \theta'_V = \tan^{-1}\frac{V_y}{V_x} \text{ if } V_x \neq 0, \text{ and}$$

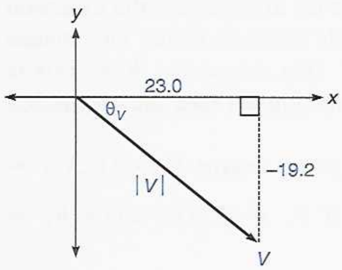
$$\theta_V = \begin{cases} \theta'_V & \text{if } V_x > 0 \\ \theta'_V - 180° \text{ if } \theta'_V > 0 & \\ \theta'_V + 180° \text{ if } \theta'_V < 0 & \text{if } V_x < 0 \end{cases}$$

Note If $V_x = 0$ then θ_V is 90° if $V_Y > 0$, and $-90°$ if $V_Y < 0$. This is clear from a sketch of the vector.

The examples and the exercises will make clear why the rule works when $V_x < 0$. Example 6–3 B illustrates finding the polar form of a vector when its rectangular form is known.

■ *Example 6–3 B*

Convert to polar form.



1. The horizontal component of a force vector is 23.0 pounds to the right; its vertical component is 19.2 pounds down. Find the force vector V, to the nearest 0.1 pound.

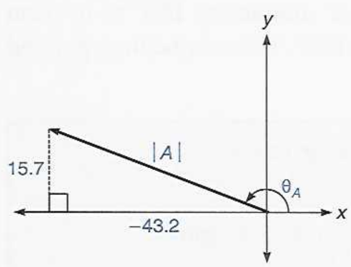
   The vector is $(23.0,-19.2)$ in rectangular form. We can find $|V|$ using the Pythagorean theorem:

$$|V|^2 = 23.0^2 + (-19.2)^2$$
$$|V| \approx 30.0$$

   We now find θ'_V the reference angle for θ_V.

$$\theta'_V = \tan^{-1}\left(\frac{-19.2}{23.0}\right) \approx -39.9°$$
$$\theta_V = \theta'_V, \quad V_x > 0$$

   Thus, $V \approx (30.0,-39.9°)$.



2. Vector $A = (-43.2,15.7)$. Find the polar form of A.

$$|A| = \sqrt{A_x^2 + A_y^2}$$
$$= \sqrt{(-43.2)^2 + (-15.7)^2} \approx 46.0 \qquad \text{Nearest tenth}$$
$$\theta'_A = \tan^{-1}\left(\frac{15.7}{-43.2}\right) \approx -20.0°$$
$$\theta_A \approx -20.0° + 180° \approx 160.0° \qquad A_x < 0,\ \theta'_A < 0$$

   Thus, $A \approx (46.0,160.0°)$. ■

Using special calculator keys

Most engineering/scientific calculators are programmed to perform the conversions of examples 6–3 A and 6–3 B. These calculators have keys marked "R → P" or simply "→P" (rectangular to polar conversion) and "P → R" or "→ R" (polar to rectangular conversion), or something equivalent. The results are stored in locations referred to as x and y. Typical keystrokes are illustrated here. Part 1 of example 6–3 A would be done as follows. (The TI-81 is discussed below.)

$A = (25.0,125°)$

25 [P → R] 125 [=] Display: [−14.33941091]

[x↔y] Display: [20.47880111]

Thus $A \approx (-14.3,20.5)$.

Part 1 of example 6–3 B would be done in the following way.

$V = (23.0,-19.2)$

23 [R → P] 19.2 [+/−] [=] Display [29.96064085]

[x↔y] Display: [−39.855454183]

Thus, $V \approx (30.0,-39.9°)$.

The TI-81 uses the values X, Y, R, θ. (Y, R, θ are $\boxed{\text{ALPHA}}$ 1, $\boxed{\times}$, and 3, respectively.) It also uses the two MATH functions ''R ▸ P('' (Rectangular to polar) and ''P ▸ R('' (Polar to rectangular).

Example 6–3 A, part 1:

$\boxed{\text{MODE}}$ Deg $\boxed{\text{ENTER}}$ Make sure the calculator is in degree mode.

$\boxed{\text{MATH}}$ 2 25 $\boxed{\text{ALPHA}}$ $\boxed{.}$ 125 $\boxed{)}$

$\boxed{\text{ENTER}}$ Display: $\boxed{-14.33941091}$

$\boxed{\text{ALPHA}}$ 1 $\boxed{\text{ENTER}}$ Display: $\boxed{20.47880111}$

Example 6–3 B, part 1:

$\boxed{\text{MATH}}$ 1 23 $\boxed{\text{ALPHA}}$ $\boxed{.}$ $\boxed{(-)}$ 19

$\boxed{.}$ 2 $\boxed{)}$ $\boxed{\text{ENTER}}$ Display: $\boxed{29.96064085}$

$\boxed{\text{ALPHA}}$ 3 $\boxed{\text{ENTER}}$ Display: $\boxed{-39.855454183}$

Addition of vectors

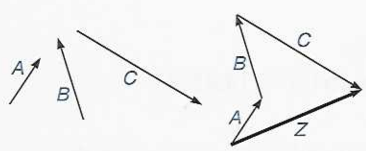


Figure 6–9

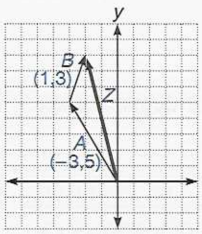
It has been shown experimentally that most natural phenomena that are described by vectors combine as if they were connected tail to head in a series. The result is a vector with its tail at the tail of the first vector in the series, and its head at the head of the last vector in the series. The resulting vector is called the **resultant vector.** This is illustrated in figure 6–9, where vectors A, B, and C combine into the resultant vector Z. This idea is the basis for the definition of addition of vectors which we develop here.

Example 6–3 C illustrates that this process is equivalent to summing all the horizontal components and, separately, the vertical components. This is the basis for the definition of the addition of two vectors.

> **Vector sum**
> Let Z be the resultant (vector sum) of two vectors $A = (A_x, A_y)$ and $B = (B_x, B_y)$. Then we say $Z = A + B$, where
> $$Z_x = A_x + B_x \quad \text{and} \quad Z_y = A_y + B_y$$

Observe that this definition describes how to add two vectors whose rectangular form is known. When vectors are known in polar form they must first be converted to rectangular form. This is illustrated in example 6–3 C.

■ Example 6–3 C



Find the vector sum of the given vectors.

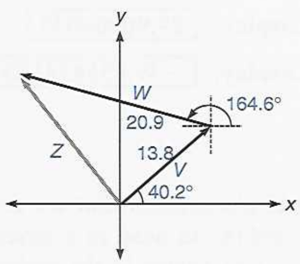
1. $A = (-3,5)$ and $B = (1,3)$

$$Z_x = A_x + B_x = -3 + 1 = -2$$
$$Z_y = A_y + B_y = 5 + 3 = 8$$
$$Z = (-2,8)$$

This is illustrated in the figure.

2. $A = (2,-\frac{1}{2})$, $B = (-6,3\frac{1}{2})$, $C = (3,-2)$

$$Z_x = A_x + B_x + C_x = 2 + (-6) + 3 = -1$$
$$Z_y = A_y + B_y + C_y = -\tfrac{1}{2} + 3\tfrac{1}{2} + (-2) = 1$$
$$Z = (-1,1)$$

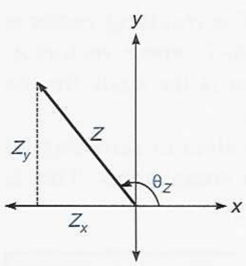


3. $V = (13.8,40.2°)$, $W = (20.9,164.6°)$

Note that these vectors are in polar form. See the figure.

$$\begin{aligned} Z_x &= V_x + W_x \\ &= |V| \cos \theta_V + |W| \cos \theta_W \\ &= 13.8 \cos 40.2° + 20.9 \cos 164.6° \approx -9.6093 \\ Z_y &= V_y + W_y \\ &= |V| \sin \theta_V + |W| \sin \theta_W \\ &= 13.8 \sin 40.2° + 20.9 \sin 164.6° \approx 14.4574 \end{aligned}$$

Thus, $Z \approx (-9.6,14.5)$ (to nearest tenth).



We should find Z in polar form, since V and W were given in this form.

$$Z = \sqrt{Z_x^2 + Z_y^2} = \sqrt{(-9.6093)^2 + 14.4589^2} \approx 17.4 \qquad \text{(nearest 0.1)}$$

$$\theta'_Z = \tan^{-1}\frac{Z_y}{Z_x} \approx \tan^{-1}\left(\frac{14.4589}{-9.6093}\right) \approx -56.4°$$

$$\theta_Z = \theta'_Z + 180° \approx 123.6° \qquad \text{Since } Z_x < 0 \text{, and } \theta'_z \text{ is negative, we add } 180°$$

Thus, in polar form, $Z \approx (17.4,123.6°)$.

4. Three forces are acting on a point, where F_1 is 25 pounds acting in the direction 30°, F_2 is 40 points acting in the direction 100°, and F_3 is 50 pounds acting in the direction −40°. Find the resultant force acting on the point.

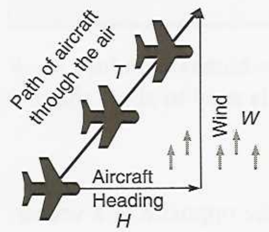
Find the resultant of the three vectors $A(25,30°)$, $B(40,100°)$, and $C(50,-40°)$.

$$\begin{aligned} Z_x &= F_{1x} + F_{2x} + F_{3x} \\ &= 25 \cos 30° + 40 \cos 100° + 50 \cos(-40°) \approx 53.01 \\ Z_y &= F_{1y} + F_{2y} + F_{3y} \\ &= 25 \sin 30° + 40 \sin 100° + 50 \sin(-40°) \approx 19.75 \\ |Z| &= \sqrt{Z_x^2 + Z_y^2} \\ &= \sqrt{53.01^2 + 19.75^2} \approx 56.6 \text{ pounds} \end{aligned}$$

$$\tan \theta'_Z = \frac{Z_y}{Z_x} = \frac{19.75}{53.01};\ \theta'_Z \approx 20.4°$$

Since Z_x and Z_y are both positive, the resultant is in the first quadrant, so $\theta_Z = \theta'_Z \approx 20.4°$.
Thus, $Z \approx (56.6, 20.4°)$. ■

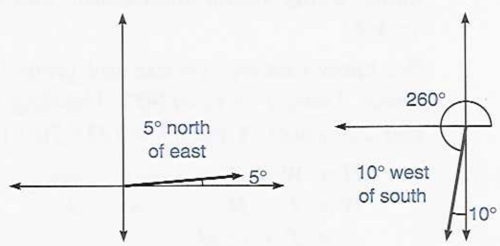
Problem 76 in the exercises presents a program that adds two or more vectors given in polar form.



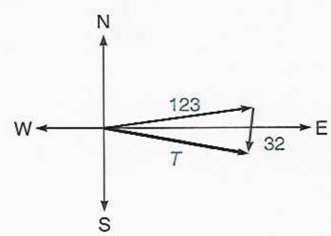
Example 6–3 D illustrates a general principle of navigating aircraft (and, analogously, ships at sea). The speed of the aircraft relative to the air is called the airspeed, and the direction in which the aircraft is pointed is its heading. The airspeed and heading combine into the heading vector, H. The wind vector, W, is the speed and direction of the wind. If we add these two vectors we get the true course and ground speed of the plane, T = H + W. T is the speed and direction relative to the earth's surface.

■ ***Example 6–3 D***

An aircraft is flying with heading 5° north of east and airspeed 123 knots. The wind is blowing at 32 knots in the direction 10° west of south. Find the true course and ground speed of the aircraft (see the figure).



$$H = (123, 5°);\ W = (32, 260°)$$



$$\begin{aligned}
T &= H + W \\
T_x &= H_x + W_x \\
&= |H|\cos\theta_H + |W|\cos\theta_W \\
&= 123\cos 5° + 32\cos 260° \approx 116.98 \\
T_y &= H_y + W_y \\
&= |H|\sin\theta_H + |W|\sin\theta_W \\
&= 123\sin 5° + 32\sin 260° \approx -20.79 \\
\theta'_T &= \tan^{-1}\frac{T_y}{T_x} = \tan^{-1}\left(\frac{-20.79}{116.98}\right) \approx -10°
\end{aligned}$$

To nearest degree

$\theta_T = \theta'_T$ since $T_x > 0$, $|T| = \sqrt{T_x^2 + T_y^2} = \sqrt{116.98^2 + (-20.79)^2} \approx 119.$

Thus, the ground speed of the aircraft is 119 knots and the true course is 10° south of east. ■

The zero vector and the opposite of a vector

It is useful to define a zero vector and the opposite of a vector. The zero vector is defined so its length is zero. The direction does not matter. The opposite of a vector is defined to have equal length but opposite direction. We do this by adding or subtracting 180°. Both definitions are in terms of polar form.

Zero vector
The vector $0 = (0,\theta)$, where θ is any angle, is the zero vector.

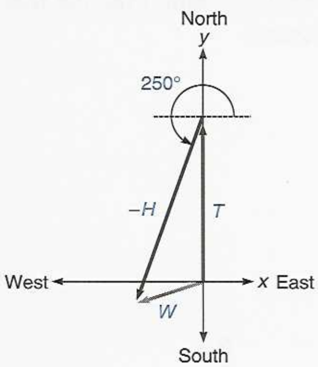
Opposite of a vector
Given a vector $V = (|V|,\theta_V)$, then $-V$ means its opposite, and $-V = (|V|,\theta_V \pm 180°)$.

In computing the opposite we generally choose whichever value, $\theta_V + 180°$ or $\theta_V - 180°$, has the smallest absolute value. It is easy to show that for any vector V,

$$V + (-V) = 0$$

Example 6–3 E illustrates uses of the zero vector and the opposite of a vector.

Example 6–3 E



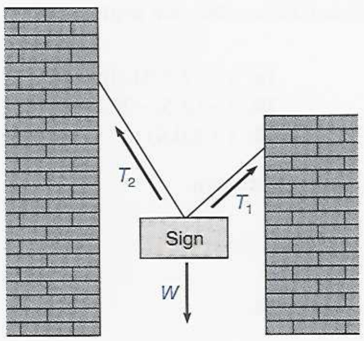
1. An aircraft's on-board inertial navigation computer shows that the aircraft is traveling due north at 200 knots with respect to the ground, and that the aircraft is headed 20° east of north with an airspeed of 225 knots. Using vector subtraction, find the wind vector W. Interpret this vector.

   We know that true course and ground speed T are due north and 200 knots. Thus, $T = (200,90°)$. Heading and airspeed are 20° east of north and 225 knots. Thus, $H = (225,70°)$ (see the figure).

   $H + W = T$ Aircraft heading + wind = true course
   $W = T - H$ Solve[1] for W
   $= T + (-H)$
   $W = (200,90°) + (225,250°)$ $-H = (225,70° + 180°)$
   $W_x = 200 \cos 90° + 225 \cos 250° \approx -76.95$
   $W_y = 200 \sin 90° + 225 \sin 250° \approx -11.43$
   $|W| = \sqrt{W_x^2 + W_y^2} \approx 77.80$
   $$\theta'_W \approx \tan^{-1}\left(\frac{-11.43}{-76.95}\right) \approx -8.4°$$
   $\theta_W = \theta'_W - 180° \approx -8.4° + 180°$ $W_x < 0$ and $\theta'_W > 0$
   $\approx -171.6°$

   Thus, $W \approx (77.8,-171.6°)$. This tells us that the wind is blowing in a direction 8.4° south of west at 77.8 knots.

[1]We are assuming we can solve a vector-valued equation as we solve real-valued equations. In fact, it can be proved that this is valid.



2. A large sign is suspended between two buildings by two wires, as in the figure. One wire acts at an angle of 45° above the horizontal and has a tension (force) T_1, of 400 pounds. If the sign weighs 800 pounds (vector W), compute a vector that describes T_2, the tension and direction of the second wire, to the nearest unit.

We use a fact from physics to describe the situation. Since the sign is motionless, all the forces acting on it must be balanced, or add to zero. Thus, we proceed as follows.

$T_1 + T_2 + W = 0$ All forces balanced

$T_2 = -T_1 - W$ Solve for T_2

$T_1 = (400{,}45°)$, so $-T_1 = (400{,}45° + 180°) = (400{,}225°)$

$W = (800{,}270°)$, so $-W = (800{,}270° - 180°) = (800{,}90°)$

$$\begin{aligned}T_{2x} &= -T_{1x} + (-W_x)\\ &= 400 \cos 225° + 800 \cos 90° \approx -282.84\\ T_{2y} &= -T_{1y} + (-W_y)\\ &= 400 \sin 225° + 800 \sin 90° \approx 517.16\\ |T_2| &= \sqrt{(-282.84)^2 + 517.16^2} \approx 589 \text{ pounds.}\\ \theta'_{T_2} &\approx \tan^{-1}\left(\frac{517.16}{-282.84}\right) \approx -61.3°\\ \theta_{T_2} &= 180° + \theta'_{T_2} \qquad T_{2x} < 0,\ \theta'_{T_2} < 0\\ &\approx 118.7°\end{aligned}$$

Thus, to the nearest unit $T_2 \approx (589{,}119°)$. ■

Mastery points

Can you

- Find the horizontal and vertical components of a vector?
- Find the magnitude and direction of a vector when given the horizontal and vertical components?
- Add or subtract vectors?
- Apply vectors to navigation and force problems?

Exercise 6–3

Convert each vector from its polar to its rectangular form. Leave all answers to the nearest tenth unless the reference angle is 30°, 45°, or 60°.

1. (40,30°) **2.** (15.2,33.6°) **3.** (100.0,122.3°) **4.** (4.2,97.3°)
5. (10.0,200.0°) **6.** (18,120°) **7.** (25,300°) **8.** (82.0,341.9°)
9. (6,−45°) **10.** (5.9,59.2°) **11.** (7.8,−264.3°) **12.** (20.0,−333.0°)

Convert each vector to polar form. Round to the nearest tenth unless an exact form is possible (if the reference angle is 30°, 45°, or 60°).

13. $(3.0,4.0)$
14. $(31.2,6.9)$
15. $(-3.0,5.2)$
16. $(-12.5,31.0)$
17. $(\sqrt{3},-2)$
18. $(3,-3)$
19. $(5,-10)$
20. $(-12.5,-20.3)$
21. $(-6.8,3.4)$
22. $(-8,4)$
23. $(-\sqrt{2},-\sqrt{8})$
24. $(-1,0.4)$

Add the following vectors. Leave the resultant in rectangular form. Round the resultant to the nearest tenth.

25. $(-3,8), (2,12)$
26. $(4,3\frac{1}{2}), (-1,-2\frac{1}{2})$
27. $(\sqrt{2},5), (\sqrt{8},1), (\sqrt{50},-6)$
28. $(5,-13), (-\frac{1}{2},8), (7\frac{1}{2},-1)$

Add the following vectors. Leave the resultant in polar form. Round the resultant to the nearest tenth.

29. $(30.0,30°), (15.2,33.6°)$
30. $(100,0°), (4.2,97.3°)$
31. $(10.0,200°), (29.3,250°)$
32. $(13.7,300.0°), (82.0,341.9°)$
33. $(3.2,-45.0°), (5.9,-59.2°)$
34. $(37.9,-100.5°), (69.2,-170.1°)$
35. $(41.9,-213.0°), (7.7,-264.3°)$
36. $(20.0,-333.0°), (39.2,-359.0°)$
37. $(3.5,19.2°), (2.7,83.1°), (4.3,145.7°)$
38. $(7.1,13.8°), (6.2,131°), (10.4,215°)$
39. $(15.3,311°), (20.9,117°), (13.2,83°)$
40. $(3.2,19.5°), (5.1,45.0°), (6.0,180°)$
41. $(3.5,-25°), (6.8,25°), (4.2,50°)$
42. $(25,-30°), (25,-60°), (25,-100°)$

43. An aircraft is moving in the direction 25° north of west at 150 knots. Find the east-west and north-south components of its velocity to the nearest knot and interpret the results.

44. An aircraft is moving in the direction 35° east of south at 120 knots. Find the east-west and north-south components of its velocity to the nearest knot and interpret the results.

45. An aircraft is moving in the direction 15° north of east at 200 knots. Find the east-west and north-south components of its velocity to the nearest knot and interpret the results.

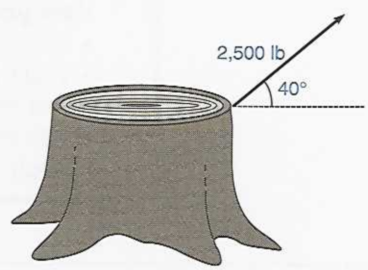
46. A rocket is climbing with a speed of 825 knots and an angle of climb of 58.6°. (The angle of climb is the angle measured from the ground to its flight path.) Find the horizontal and vertical components of the rocket's velocity, to the nearest knot.

47. An aircraft is traveling in a direction 30° west of north. Its speed is 456 knots. Find the east-west and north-south components of its velocity, to the nearest knot. (Remember that 30° west of north corresponds to the angle 120°.)

48. At the location of a ship, the Gulf Stream ocean current is moving to the northwest at a speed of 8.2 knots. Find the east-west and north-south components of its velocity, to the nearest knot.

49. A ship has left an east-coast harbor and has been sailing in a direction 32° north of east for 2.5 hours, at a speed of 18 knots. (a) How far north of the harbor has it gone, to the nearest nautical mile? (b) How far east of the harbor has it gone, to the nearest nautical mile?

50. A force is acting on a tree stump at a 40° angle of elevation. See the diagram. If the force is 2,500 pounds, find its vertical and horizontal components, to the nearest pound.



51. A 2,250 pound force is pulling on a sled loaded with lumber, at an angle of elevation of 33°. If the sled will not move until the horizontal component of the force exceeds 1,900 pounds, will the sled move?

52. A sled loaded with lumber will not move until the horizontal component of the applied force is 1,200 pounds or more. If a winch being used to move the sled can apply a maximum force of 1,700 pounds, what is the largest angle of elevation at which the winch can act on the sled and move it?

53. Consider a force of 1,000 pounds acting at an angle of elevation of 15° on a point.

a. Compute the horizontal and vertical components of the force.

b. Double the force to 2,000 pounds and recompute the horizontal and vertical components. Do they double also?

c. Double the angle of elevation to 30° (keep the force at 1,000 pounds). Recompute the horizontal and vertical components. Do they double also?

54. A plane is flying over Minneapolis with a ground speed of 200 miles per hour and true course due east. After 1 hour it turns to a true course of 60° south of east, maintaining the same ground speed. After flying for an additional half-hour, the navigator notes its position on a map. How far and in what direction is the plane from Minneapolis, to the nearest unit?

55. A plane is flying over Orlando with ground speed 135 miles per hour, and true course 23° north of east. After 1 hour it turns to a true course of 40° south of west, maintaining the same ground speed. After flying for an additional hour the navigator notes its position on a map. How far and in what direction is the plane from Orlando, to the nearest unit?

56. Two forces are acting on a point, 12.6 pounds in the direction 123° and 15.8 pounds in the direction 211°. Compute the magnitude and direction of the resultant force to the nearest tenth.

57. Two forces are acting on a point, 2.6 newtons in the direction 18.3° and 15.8 newtons in the direction −86.2°. Compute the magnitude and direction of the resultant force, to the nearest tenth.

58. Three forces are acting on a point: 27.6 newtons in the direction 18.3°, 32.1 newtons at 223.0°, and 46.8 newtons at −30.0°. Find the resultant force acting on the point, to the nearest 0.1 newton.

59. Three forces are acting on a point: 199 pounds at 19.0°, 175 pounds at 131.0°, and 96 pounds at 130.0°. Find the resultant force acting on the point, to the nearest 0.1 pound.

60. A ship leaves its harbor traveling 10° north of east. After 1 hour it turns to the direction 40° south of east. After 2 more hours, it turns to the direction 15° west of south. The shop travels for 1 half hour more and then stops. The ship has maintained a steady speed of 16 knots (nautical miles per hour) for the entire trip. How many nautical miles, and in what direction, is the ship from its starting position, to the nearest knot?

61. A ship leaves its harbor traveling 15° west of south. After 1 hour it turns to the direction 34° south of west. After 2 more hours, it turns to the direction 10° north of west. The ship travels for 1 half hour more and then stops. The ship has maintained a steady speed of 20 knots for the entire trip. How many nautical miles, and in what direction, is the ship from its starting position, to the nearest knot?

62. An aircraft is flying with an airspeed of 123 knots and a heading of 30° west of north. The wind is blowing in the direction 15° south of west at 26 knots. Add the heading and wind vectors to find the aircraft's true course and ground speed, to the nearest integer.

63. If the wind in problem 62 now shifts to 35 knots in the direction 10° west of south, find the aircraft's new true course and ground speed, to the nearest integer.

64. A ship is traveling through an ocean current that flows in the direction 5° east of north at 7.2 knots. The ship's heading is 10° north of west, and its speed relative to the water is 19.6 knots. Add these two vectors to find the ship's true course and speed, to the nearest tenth.

65. A ship is traveling through an ocean current that flows in the direction 15° west of north at 7.2 knots. The ship's heading is 10° north of east, and its speed relative to the water is 26.1 knots. Add these two vectors to find the ship's true course and speed, to the nearest tenth.

66. The voltage in an alternating current circuit adds vectorially. If one voltage E_1 is 122 volts with phase angle 30° and a second voltage E_2 is 86 volts with phase angle 21°, find the magnitude and phase angle of the resultant voltage E_T to the nearest unit.

67. (Refer to problem 66.) In an AC circuit E_1 is 240 volts at −45° and E_2 is 115 volts at +45°. Find the resultant E_T to the nearest unit.

68. An aircraft has a ground speed of 135 knots and true course 35° east of north. If its heading is due north and airspeed is 120 knots, find the direction and speed of the wind, to the nearest unit.

69. An aircraft has a ground speed of 80 knots and true course 15° west of north. If the wind is directly from the northeast at 12 knots, find the plane's heading and airspeed, to the nearest unit.

70. A ship is traveling at 14 knots, relative to the water, with heading 20° south of west. If the true speed and direction of the ship is 12 knots due west, find the speed and direction of the ocean current, to the nearest tenth.

71. The ocean current in a certain area is 6.4 knots with direction 8° east of south. A ship in the area is traveling with true direction of 12 knots at 25° east of north. Find the ship's heading and speed relative to the water, to the nearest tenth.

72. Two cables support a 1-ton (2,000 pound) sign between two buildings. One of the cables has a tension of 1,500 pounds and acts at an angle of 33° above the horizontal. The other cable is attached to the other building. Find the tension in the other cable as well as its direction relative to the horizontal, to the nearest unit.

73. Two cables support a sign between two buildings. The tension and direction of one cable is 456 pounds at 63° above the horizontal. If the sign weighs 650 pounds, find the tension in the other cable, as well as its direction relative to the horizontal, to the nearest unit.

74. Prove that vector addition is commutative. That is, if A and B are vectors, then $A + B = B + A$. (*Hint:* Real number addition is commutative, and the horizontal and vertical components of a vector are real values.)

75. Prove that vector addition is associative. That is, if A, B, and C are vectors, then $(A + B) + C = A + (B + C)$. (*Hint:* Real number addition is associative, and the horizontal and vertical components of a vector are real values.)

76. Write a program for a computer or programmable calculator that will add two or more vectors when given in polar form.

Chapter 6 summary

- **The law of sines**
  In any triangle ABC, $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$.
- **The ambiguous case** When the law of sines is applied to a situation in which only one of the two angles of a triangle is known, it is possible to get two different solutions to the problem. This is called the ambiguous case.
- **The law of cosines** For any triangle ABC,
  $a^2 = b^2 + c^2 - 2bc \cos A$
  $b^2 = a^2 + c^2 - 2ac \cos B$
  $c^2 = a^2 + b^2 - 2ab \cos C$
- **Polar form of a vector** A vector A in polar form is the ordered pair $A = (|A|,\theta_A)$.
  $|A|$ is the magnitude of vector A; $|A| \geq 0$.
  θ_A is the direction of vector A.
- **Rectangular form of a vector** A vector A in rectangular form is $A = (A_x,A_y)$, where A_x is the horizontal component of the vector and A_y is the vertical component of the vector.
- **To convert polar form to rectangular form**
  Given vector $A = (|A|,\theta_A) = (A_x,A_y)$,
  $$A_x = |A| \cos \theta_A$$
  $$A_y = |A| \sin \theta_A$$
- **To convert rectangular form to polar form**
  Given vector $V = (V_x,v_y) = (|V|,\theta_V)$. Then,
  $|V| = \sqrt{V_x^2 + V_y^2}$, $\theta_V' = \tan^{-1}\dfrac{V_y}{V_x}$ if $V_x \neq 0$, and
  $$\theta_V = \begin{cases} \theta_V' & \text{if } V_x > 0 \\ \theta_V' - 180° \text{ if } \theta_V' > 0 & \\ \theta_V' + 180° \text{ if } \theta_V' < 0 & \text{if } V_x < 0 \end{cases}$$
- **Vector sum** Let $Z = (Z_x,Z_y)$ be the resultant (vector sum) of two vectors $A = (A_x,A_y)$ and $B = (B_x,B_y)$. Then, $Z = A + B$, and $Z_x = A_x + B_x$ and $Z_y = A_y + B_y$.
- With regard to an aircraft, $T = H + W$, where
  T is the vector of ground speed and true course
  H is the vector of airspeed and aircraft heading
  W is the vector of the speed and direction of the wind.
- **Zero vector** The vector $0 = (0,\theta)$, where θ is any angle, is the zero vector.
- **Opposite of a vector** Given a vector $V = (|V|,\theta_V)$, then $-V = (|V|,\theta_V \pm 180°)$.

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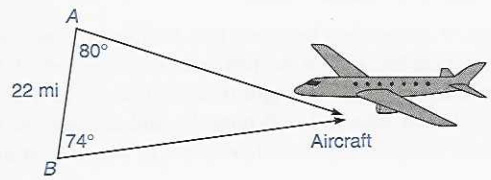
Chapter 6 review

[6–1] Solve the following oblique triangles using the law of sines. Round answers to the nearest tenth.

1. $a = 10.6, A = 47.9°, B = 10.3°$
2. $b = 3.55, B = 23.8°, C = 5.2°$
3. $a = 10.0, b = 13.0, B = 79.0°$
4. $a = 12.6, c = 7.0, C = 32.7°$
5. A lost pilot is given a position report by triangulation with two radar sites. The situation is shown in the diagram. Find the distance of the aircraft from radar site A, to the nearest mile.



[6–2] Solve the following oblique triangles, to the nearest tenth. You will have to use the law of cosines as the first step.

6. $a = 4.1, b = 6.8, C = 29.4°$
7. $b = 60.0, c = 20.0, A = 92.1°$
8. $a = 21.4, c = 27.0, B = 112°$
9. $a = 43.5, b = 17.8, c = 35.0$
10. $a = 31.7, b = 80.0, c = 105$

11. A technician is setting up a numerically controlled grinding machine. A triangular pattern is to be ground and, therefore, must be coordinatized. The vertices of the triangle are at $A(-2,5)$, $B(4,7)$, and $C(5,-2)$. Solve the resulting triangle. Round answers to the nearest 0.1.

12. A ship leaves a harbor heading due west and travels 23.3 km. It then turns north through a 63° angle and travels for another 10.0 km. How far is the ship from its starting point, to the nearest kilometer?

[6–3]

13. Convert the vector (27.2,29.0°), to rectangular form. Round to the nearest 0.1.

14. The horizontal and vertical components of a vector are 19.6 and 30.5, respectively. Find the magnitude and direction of the vector.

15. A rocket climbs with a speed of 450 knots and an angle of climb of 34.6°. (The angle of climb is the angle measured from the ground to its flight path.) Find the horizontal and vertical components of the rocket's velocity, to the nearest knot.

16. A force is acting on a cart at a 13.5° angle of elevation (13.5° measured from the ground up to the force vector). If the force is 256 pounds, find the vertical and horizontal components, to the nearest pound.

Add the following vectors. Round the resultant to the nearest tenth.

17. (33.0,14.7°), (15.2,33.6°)
18. (10.2,112.3°), (4.2,19.3°)
19. (3.5,29.2°), (1.7,43.1°), (4.3,115.0°)
20. (7.1,13.8°), (6.6,142.0°), (11.9,215.0°)

21. Two forces are acting on a point, 126 pounds in the direction 223° and 158 pounds in the direction 311°. Compute the magnitude and direction of the resultant force, to the nearest unit.

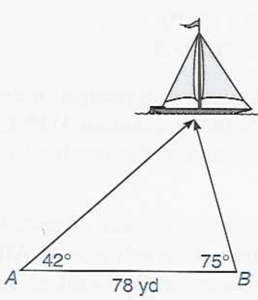
22. A ship leaves its harbor traveling 15° south of east. After 1 hour it turns to the direction 40° south of east. After 2 more hours it turns to the direction 25° west of south. The ship travels for 1 half-hour more and then stops. The ship has maintained a steady speed of 18 knots for the entire trip. How many nautical miles and in what direction is the ship from its starting position, to the nearest integer?

23. An aircraft is flying with an airspeed of 195 knots and heading of 24° west of north. The wind is blowing in the direction 25° west of south at 19 knots. Add the heading and wind vectors to find the aircraft's true course and ground speed, to the nearest integer.

24. An aircraft's true course and ground speed are 120° at 225 knots. The wind is blowing in a direction 15° south of east at 30 knots. Find the heading and airspeed of the aircraft, to the nearest unit.

Chapter 6 test

1. In triangle ABC, $b = 22.6$, $A = 13.5°$, and $C = 82.1°$. Solve this triangle. Round answers to the nearest tenth.
2. In triangle ABC, $b = 22.6$, $c = 24.0$, and $C = 62.1°$. Solve this triangle. Round answers to the nearest tenth.
3. In triangle ABC, $a = 25.9$, $c = 16.2$, and $B = 100°$. Solve this triangle. Round answers to the nearest tenth.
4. In triangle ABC, $a = 2.55$, $b = 3.12$, and $c = 4.00$. Solve this triangle. Round answers to the nearest tenth.
5. The distance to a boat on a lake is being found by triangulation from two points on shore. The situation is shown in the diagram. Find the distance to the boat from site B, to the nearest yard.



6. Given the three points $A(6,8)$, $B(-3,5)$, and $C(10,-4)$, find the angle formed by line segments AB and BC, to the nearest $0.1°$.
7. Convert the vector $(2,30°)$ to rectangular form. Leave the answer in exact form.
8. The horizontal and vertical components of a vector are 4.0 and 5.0. Find the magnitude and direction of the vector, to the nearest tenth.
9. Add the vectors $(5.4,19.0°)$ and $(8.0,123°)$. Round the resultant to the nearest tenth.
10. A sign is suspended between two buildings. One cable from which the sign is suspended has a tension of 535 pounds and acts at an angle of elevation of 62°. If the weight of the sign is 1,000 pounds, find the tension and angle of elevation in the other cable, to the nearest unit.

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